

# Interaction Protected Topological Insulators with Time Reversal Symmetry

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Anderson's localization on the edge of two dimensional time reversal (TR) topological insulator (TI) is studied. For the non-interacting case the topological protection acts accordingly to the  $\mathbb{Z}_2$  classification, leading to conducting and insulating phases for odd and even fillings respectively. In the presence of repulsive interaction the phase diagram is notably changed. We show that for sufficiently strong values of the interaction the zero temperature fixed point of the TI is conducting, including the case of even fillings. We compute the boundaries of the conducting phase for various fillings and types of disorder.

## I. INTRODUCTION

Time reversal non-interacting TIs are realized in materials with strong spin orbit interaction in two<sup>1-3</sup> and three<sup>4-8</sup> dimensions. Also known as quantum spin Hall insulators, these materials have an insulating bulk, while hosting gapless surface states.

For the non-interacting case the classification of disordered TIs is complete<sup>9-13</sup>. In two dimensions, non interacting TR invariant TIs are classified by a  $\mathbb{Z}_2$  topological invariant, according to the number of helical edge states<sup>14,15</sup>. The back scattering by TR invariant disorder is possible only between states that are not Kramer's partners. Therefore for an odd number of the helical states the conducting state is protected. For an even number of helical edge modes the electrons can be localized completely by scattering among non-Kramers pairs. This state is therefore equivalent to a trivial insulator in agreement with a more general Haldane criterion<sup>16</sup>.

Interacting integer and fractional TI's are a subject of active research. Interaction may lead to strongly correlated ground states<sup>17-21</sup> with fractional excitations and non trivial statistics<sup>22-24</sup>. The classification of TI's in the interacting case is yet unknown. An approach, based on thermal response and its relation to a quantum anomaly, valid beyond single particle picture, was proposed in Ref.<sup>25</sup>.

It is commonly accepted, that the charge transport in the ideal TI occurs via protected a single helical edge mode, with the universal quantized conductance  $2e^2/h$ . In reality the conductance differ from this value due to back scattering processes. The latter may occur via the combination of the two electron scattering and the disorder potential<sup>26,27</sup>, coupling to the bulk via electron puddles<sup>28</sup> or due to the magnetic impurities. In the later case the interaction stabilizes the conducting phase, and the quantum phase transition as function of Luttinger liquid (LL) parameter is predicted for the fractional TIs<sup>30</sup>.

In this work we study the localization by TR disorder on the edge states of a TI in the presence of repulsive interaction. Although we focus on the TI's at integer fillings ( $\nu$ ), that in the absence of the disorder possess  $\nu$  helical edge states, the same analysis applies for narrow stripe of TI at  $\nu = 1$ .<sup>29</sup> The inclusion of TR disorder

drives the non-interacting system to a state with a single or no helical edge states. We show that the presence of the repulsive interaction can stabilize the conducting phase.

We model the disorder by a short range static potential that due to the spin orbit interaction mixes different helical states, except those that are connected by the TR symmetry. We consider a generic finite range interaction between the electrons, with all possible matrix elements allowed by symmetry. We consider the case of a single impurity and the random disorder, with a scattering length shorter than the sample size. We perform one loop renormalization group (RG) analysis, analogous to Kane-Fisher<sup>31</sup> and Giamarchi-Schultz<sup>32</sup> study of localization in one dimensional systems.

Our analysis shows that the low energy fixed point is determined by the magnitude of the interaction and its effective radius. For interaction stronger than some critical value the low temperature phase is conducting.

## II. THE MODEL

The appearance of the helical edge states can be understood on the level of non interacting electrons. In the presence of a Rashba spin-orbit (SO) interaction the single particle Hamiltonian is given by

$$H = \frac{\hat{p}_x^2 + \hat{p}_y^2}{2m_e} + \alpha_{SO}(\vec{p} \times \vec{\sigma}) \cdot \nabla V(x, y) + V(x, y), \quad (1)$$

where  $m_e$  is effective mass of an electron and  $\alpha_{SO}$  is the strength of the SO coupling. For the parabolic potential  $V(y) = y^2/2m_e\alpha_{SO}^2$ , the Hamiltonian (1)

$$H = \frac{\hat{p}_y^2}{2m_e} + \frac{1}{2m_e} \left( \hat{p}_x - \frac{y}{\alpha_{SO}} \sigma_z \right)^2 \quad (2)$$

corresponds to two replicas of fermions subject to opposite magnetic fields  $|B| = \alpha_{SO}^{-1}$ . For the integer fillings,  $\nu = \alpha_{SO}^{-1}A/\Phi_0$  ( $A$  being an area of sample and  $\Phi_0 = hc/e$  the flux quantum) the bulk forms an incompressible state with a gap of size  $\hbar\alpha_{SO}^{-1}/m_e$ .

An addition of a smooth confining potential curves the Landau levels, as shown in Fig.1, leading to  $\nu$  gapless

helical edge states<sup>14,15</sup>. Assuming that the single particle gap formed in the bulk is not closed the effects of interaction can be taken into account within the helical edge states. This phenomenological approach can be microscopically justified within the sliding Luttinger liquid model<sup>19,21</sup>. However, the resulting helical edge description is believed to be the correct low energy model, valid beyond the sliding LL approximation.

To account for the interaction, it is natural to pass to the bosonic description, defining bosonic fields  $R_i/L_i$ , related to the right/left density components by  $\rho_{R,i} = \partial_x R_i/2\pi$  and  $\rho_{L,i} = -\partial_x L_i/2\pi$ <sup>33</sup>. These fields satisfy the canonic commutation relations  $[R_j(x), R_{j'}(y)] = -[L_j(x), L_{j'}(y)] = i\pi \text{sgn}(x-y)\delta_{jj'}$ . The electronic operators are represented as  $\psi_{R,j} = e^{-iR_j}/\sqrt{2\pi a}$ , and  $\psi_{L,j} = e^{-iL_j}/\sqrt{2\pi a}$ , with  $a$  a short distance cut-off.

In the absence of the Umklapp and  $2k_F$  electron-electron scattering the interaction between  $\nu$  modes, consistent with TR symmetry, is represented by the following action

$$S = \frac{1}{2\pi} \int dx dt \left( \partial_x \Phi^T \mathcal{K} \partial_t \Phi - \partial_x \Phi^T \mathcal{M} \partial_x \Phi \right), \quad (3)$$

where we use the compact notations  $\Phi = (\mathbf{R}, \mathbf{L})$ ,  $\mathbf{R} = (R_1, R_2, \dots, R_\nu)$  and similarly for  $\mathbf{L}$ . Here the matrix  $\mathcal{K}$  encodes the commutation relations of the fields and can be written as  $\mathcal{K} = \sigma_z \otimes \mathbb{I}_\nu$ , where the  $\sigma_z$  is a Pauli matrix that acts in the right/left movers subspace while  $\mathbb{I}_\nu$  is the identity matrix in the space of  $\nu$  modes. The positive definite matrix  $\mathcal{M}$  accounts for interaction. The helical edge modes are separated in space by a distance  $d$ .

The symmetry under TR requires that  $\{T, \mathcal{K}\} = [T, \mathcal{M}] = 0$ , where  $T = \sigma_x \otimes \mathbb{I}_\nu$  is a time reversal symmetry operator. This restricts the interaction matrix  $\mathcal{M}$  to the form

$$\mathcal{M} = \begin{bmatrix} M_{\text{fw}} & M_{\text{bw}} \\ M_{\text{bw}} & M_{\text{fw}} \end{bmatrix} = \mathbb{I}_2 \otimes M_{\text{fw}} + \sigma_x \otimes M_{\text{bw}}, \quad (4)$$

where  $(M_{\text{fw}})_{ij}$  describes the *forward* interaction between the copropagating modes  $\rho_{R,i}$  and  $\rho_{R,j}$  (similarly for left movers).  $(M_{\text{bw}})_{ij}$  is an element of a symmetric matrix in the channel space that describes the *backward* interaction between  $R_i$  and  $L_j$ . We assume that the interaction between helical modes  $i$  and  $j$  is translationally invariant, and depends only on the relative distance  $|i-j|$ . In the absence of disorder the spectrum of this model is gapless.

The presence of impurities may dramatically change the states of edge modes. TR invariant disorder mixes helical states that belong to different Kramer's pairs and induces backward scattering processes. We consider two cases: (a) single impurity scattering and (b) random disorder. Single impurity scattering is the dominant process if the mean free path of the electrons is larger than the sample size. In the opposite case (the electrons' mean free path is smaller than the sample size) the localization is dominated by multiple scattering.

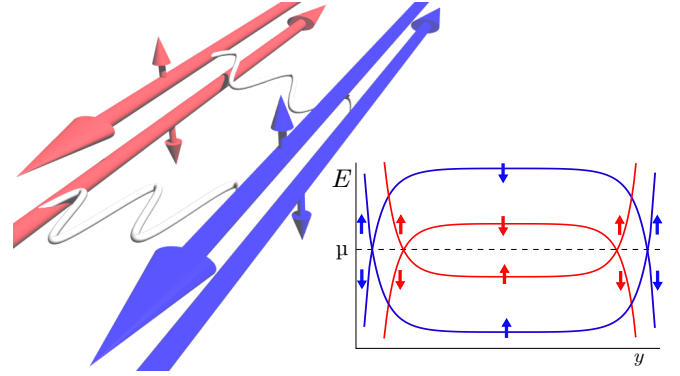


FIG. 1. (color on line) Helical edge modes. The wavy lines represent the back-scattering events between non-Kramers pairs (in red and blue). Insert.- Schematic band structure of a TI.

### A. Single Impurity

We analyze the single impurity case first. The single impurity, located on the edge backscatter between states  $i$  and  $j$  that are not connected by TR symmetry,  $\mathcal{O}_{ij}^{\text{imp}} = \mu_{ij} \psi_{R,i}^\dagger(0) \psi_{L,j}(0)$ , where  $\mu_{ij}$  is proportional to the impurity potential at  $k = 2k_F$ . The renormalization of the strength  $\mu_{ij}$  is a straightforward generalization of standard Kane-Fisher analysis<sup>31</sup>

$$\frac{d\mu_{ij}}{dl} = (1 - \Delta_{ij})\mu_{ij}. \quad (5)$$

Here  $\Delta_{ij}$  is the scaling dimension of the scattering process  $\langle \psi_{R,i}^\dagger(\tau) \psi_{L,j}(\tau) \psi_{L,j}^\dagger(0) \psi_{R,i}(0) \rangle \sim |\tau|^{-2\Delta_{ij}}$ . From Eq. (5) it follows that a single impurity is an irrelevant perturbation if  $\Delta_{ij} > 1$ .

The scaling dimension  $\Delta$  is controlled by the backward interaction matrix  $M_{\text{bw}}$ . For the simple case  $\nu = 2$  only two helical modes propagate on the edge. If the separation between the helical states is larger than the interaction radius the effective Hamiltonian is given by  $H = H_{\text{fw}} + H_{\text{bw}}$  with

$$H_{\text{fw}} = \frac{1}{2\pi} \sum_{i=1}^2 \int dx \left( v_F + \frac{g_4^0}{\pi} \right) ((\partial_x R_i)^2 + (\partial_x L_i)^2),$$

$$H_{\text{bw}} = -\frac{g_2^0}{\pi^2} \sum_{i=1}^2 \int dx \partial_x R_i \partial_x L_i. \quad (6)$$

Here  $v_F$  the Fermi velocity while  $g_4^0$  and  $g_2^0$  parameterize the forward and backward interactions respectively. Defining the fields  $\varphi_i = (R_i - L_i)/\sqrt{2}$  and  $\theta_i = (R_i + L_i)/\sqrt{2}$ , the above Hamiltonian can be written in a Luttinger liquid (LL) form

$$H = \frac{u}{2\pi} \sum_{i=1}^2 \int dx \left( (\partial_x \theta_i)^2 K + \frac{(\partial_x \varphi_i)^2}{K} \right), \quad (7)$$

where  $u = (v_F + g_4^0/\pi)\sqrt{1 - (\lambda_2)^2}$  is the renormalized sound velocity,  $\lambda_2 = \frac{g_2^0}{\pi(v_F + g_4^0/\pi)}$  and  $K = \sqrt{\frac{1 - \lambda_2}{1 + \lambda_2}}$  is the LL parameter ( $K < 1$  for repulsive and  $K > 1$  for attractive interaction within the mode). In this case the single impurity operator  $\mathcal{O}_{12}^{\text{imp}}$  has the scaling dimension  $\Delta_{12} = K/2 + 1/2K$ , so the single impurity is irrelevant for any interaction. This is in stark contrast with the non helical LL where the scaling dimension of the disorder operator is  $K$ , so the disorder is irrelevant only for an attractive interaction<sup>32</sup>. If the LL's are different the scaling dimension is controlled by both LL parameters  $\Delta_{12} = (K_1 + 1/K_1 + K_2 + 1/K_2)/4$ . This result implies that the scattering between different edge states is accompanied by the zero bias anomaly that suppresses the probability of this process. This is in contrast to Kane-Fisher case<sup>31</sup>, where the back scattering occurs in the same LL and has no zero bias anomaly suppression. For TI this processes is forbidden by the TR symmetry.

For the case where the helical edge states are located within the radius of interaction (or for any long range interaction potential) the interaction matrices (4) are

$$M_{\text{fw}} = \begin{pmatrix} v_F + g_4^0 & g_4^1 \\ g_4^1 & v_F + g_4^0 \end{pmatrix}, \quad M_{\text{bw}} = - \begin{pmatrix} g_2^0 & g_2^1 \\ g_2^1 & g_2^0 \end{pmatrix}. \quad (8)$$

The scaling dimension  $\Delta_{12}$  is

$$\Delta_{12} = \frac{1}{2} (F_1 + F_2), \quad (9)$$

where, for repulsive interactions

$$F_1 = \sqrt{\frac{1 + \lambda_4^1 - \lambda_2^0 - \lambda_2^1}{1 + \lambda_4^1 + \lambda_2^0 + \lambda_2^1}}, \quad F_1 \in [0, 1] \quad (10)$$

$$F_2 = \sqrt{\frac{1 - \lambda_4^1 + \lambda_2^0 - \lambda_2^1}{1 - \lambda_4^1 - \lambda_2^0 + \lambda_2^1}}, \quad F_2 \in [0, \infty] \quad (11)$$

with  $\lambda_a^b = \frac{g_a^b}{\pi(v_F + g_4^0/\pi)}$ . In the limit  $g_4^0 = g_2^0 = g_4^1 = 0$  of no interactions within the helical modes the scaling dimension  $\Delta_{12} = \sqrt{\frac{1 - \lambda_2^1}{1 + \lambda_2^1}} < 1$  is the same as for scattering by magnetic impurities<sup>30</sup>.

## B. Random Disorder

Now we switch to the case of multiple impurities on the edge. This perturbation is described by

$$\mathcal{O}_{ij}^{\text{dis}} = \int dx \xi_{ij}(x) (\psi_{R,i}^\dagger \psi_{L,j} - \psi_{L,i}^\dagger \psi_{R,j}). \quad (12)$$

Here  $\xi_{ij}(x)$  is the (random) scattering amplitude, and  $\xi_{ii} = 0$  due to the TR symmetry. We model the scattering to be local along the edge and uncorrelated for the different pairs of helical states

$$\langle \xi_{ij}(x) \xi_{kl}(x') \rangle = W_{ij} \delta_{ik} \delta_{jl} \delta(x - x'). \quad (13)$$

We now follow the steps of Giamarchi-Schultz renormalization group analysis<sup>32</sup>. For the weak disorder one finds<sup>32</sup>

$$\frac{dW_{ij}}{d\ell} = (3 - 2\Delta_{ij})W_{ij}, \quad (14)$$

where  $\Delta_{ij} = \Delta_{|i-j|}$  is the scaling dimension of scattering process (12) between helical states  $i$  and  $j$  allowed by TR symmetry. In the conducting phase the disorder is an irrelevant perturbation, and all  $W_{ij}$  flow to zero. This requires  $\Delta_{|i-j|} > \frac{3}{2}$  for all pairs  $i, j$ . Let us consider two limiting cases: (i) disorder that mixes only the nearest modes  $W_{ij} \sim W \delta_{i,j+1}$ ; (ii) the disorder that mixes the modes uniformly  $W_{ij} \sim W$ . All physical realizations lie in between these two limits.

The simplest situation is realized for  $\nu = 2$  where the limits (i) and (ii) coincide. In that case, the scaling of the disorder operator is given by Eq.(9). In the absence of inter-mode interaction the scaling dimension of a back scattering operator is  $\Delta_{12} = (K + K^{-1})/2$ . Therefore the system flows to a conducting fixed point for  $K < (3 - \sqrt{5})/2$ . In the presence of inter mode inter-

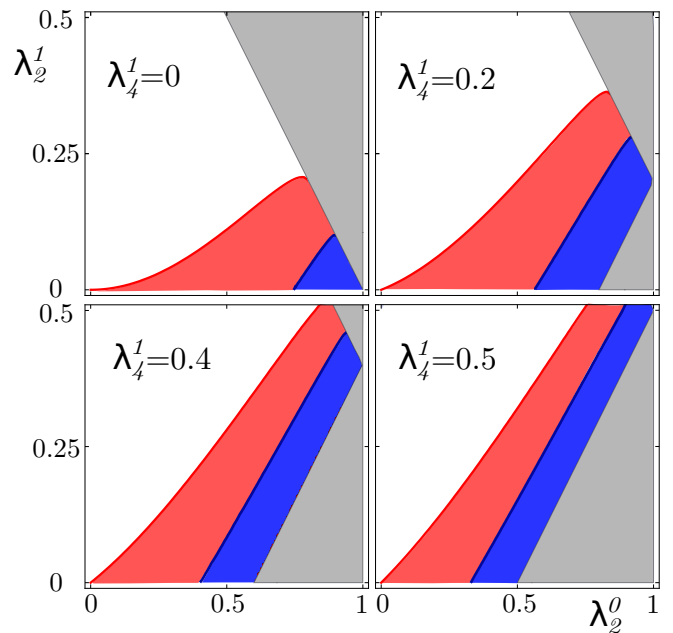


FIG. 2. (color online) Phase diagram for  $(\lambda_2^0, \lambda_2^1)$ . Red region corresponds to the conducting phase for a single impurity. Multiple impurities are irrelevant in the blue region. The gray region is forbidden by positivity of matrix  $\mathcal{M}$ .

action the phase diagram is shown in Fig. (2) as a function of interaction parameters. The symmetry between intra and inter mode forward scattering ( $\lambda_4^1 \rightarrow 1$ ) enhances the conducting phase.

### C. Effect of two particle processes

For weak interactions, two particle processes are less relevant than single particle events. For sufficiently strong interactions, they start to compete. We analyze here the following processes involving two particle events

$$\mathcal{O}_{II,c} = t_c \int dx \delta(x) \psi_{R,1}^\dagger \psi_{L,1}^\dagger \psi_{R,2} \psi_{L,2}, \quad (15)$$

$$\mathcal{O}_{II,s} = t_s \int dx \delta(x) \psi_{R,1}^\dagger \psi_{R,2}^\dagger \psi_{L,1} \psi_{L,2}, \quad (16)$$

which correspond to the transfer of  $2e$  charge and two particle backscattering respectively. These processes renormalize according to

$$\frac{dt_a}{dl} = (1 - \Delta_a) t_a, \quad (17)$$

with  $a = (s, c)$ . Here  $\Delta_a$  is the scaling dimension of the operator  $\mathcal{O}_{II,a}$ . They are

$$\Delta_c = 2F_2 \quad \text{and} \quad \Delta_s = 2F_1. \quad (18)$$

In the case of  $1/2 < F_1 < 1$ , the system is in the conducting phase for  $F_1 + F_2 > 2$ . The correction to the conductance  $G = 4e^2/h$  scales with the temperature as

$$\delta G \sim \begin{cases} -c_1 \mu_{12} \left( \frac{2\pi a T}{u} \right)^{F_1+F_2-2}, & \text{if } F_2 < 3F_1, \\ -c_2 \mu_{12}^2 v_F \left( \frac{2\pi a T}{u} \right)^{4F_1-2}, & \text{if } F_2 > 3F_1. \end{cases} \quad (19)$$

where  $c_i$  are non-universal parameters. If  $F_1 < 1/2$ , the second order process  $\mathcal{O}_{II,s}$  becomes relevant. The conductance then becomes non monotonous at large temperatures (see Fig.3)

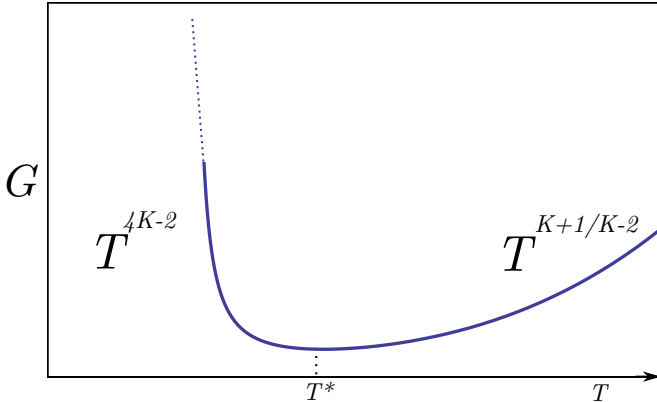


FIG. 3. (color online). Sketch of the conductance as function of temperature. For  $F_1 < 1/2$ , the two particle process  $\mathcal{O}_{II,s}$  becomes relevant, leading to a non monotonous behaviour of the conductance at high temperatures. In the figure we take  $F_1 = K$  and  $F_2 = 1/K$ , which correspond to the simplest case of interactions just within each helical mode, discussed in Eq. (6)

For random disorder, the two particle operators are

$$\mathcal{D}_{II,c} = \int dx \xi_c(x) \psi_{R,1}^\dagger \psi_{L,1}^\dagger \psi_{R,2} \psi_{L,2}, \quad (20)$$

$$\mathcal{D}_{II,s} = \int dx \xi_s(x) \psi_{R,1}^\dagger \psi_{R,2}^\dagger \psi_{L,1} \psi_{L,2}, \quad (21)$$

where the  $\xi(x)_a$  are uncorrelated random variables with  $\langle \xi_a(x) \xi_b(x') \rangle = W_a \delta_{ab} \delta(x - x')$ . These processes renormalize according to the RG equations

$$\frac{dW_a}{dl} = \left( \frac{3}{2} - \Delta_a \right) W_a, \quad (22)$$

with  $\Delta_a$  given by (18). For  $3/4 < F_1 < 1$  and  $F_1 + F_2 > 3$ , the conducting phase remains. If  $F_1 < 3/4$  (still with  $F_1 + F_2 > 3$ ), the process  $\mathcal{D}_{II,s}$  becomes relevant under RG and conductance becomes non-monotonous at high temperatures (similar to the case of single impurity).

### D. $\nu \gg 1$ Helical Edge Modes

We now proceed with a more general case of  $\nu$  helical states. To calculate  $\Delta$ , we consider the operators of the form  $\Psi_{\mathbf{m}} = e^{i\mathbf{m} \cdot \Phi}$  where each vector  $\mathbf{m} = (\mathbf{m}_R, \mathbf{m}_L)$  corresponds to a different physical process. For example

$$(\mathbf{m}_R)_i = \delta_{ik}, \quad (\mathbf{m}_L)_i = -\delta_{i,k+l}, \quad (23)$$

describes an operator  $\Psi_{\mathbf{m}}$  that backscatter a right mover in the mode  $k$  to a left mover in the mode  $k + l$ . Using the (quadratic) action (3), one computes the scaling dimension of  $\Psi_{\mathbf{m}}$

$$\Delta[\Psi_{\mathbf{m}}] = \frac{1}{2} \mathbf{m}^T \Lambda \mathbf{m}, \quad (24)$$

where  $\Lambda = \mathcal{M}^{-\frac{1}{2}} |\mathcal{M}^{\frac{1}{2}} \mathcal{K} \mathcal{M}^{\frac{1}{2}}| \mathcal{M}^{-\frac{1}{2}}$ . Here the absolute value of a matrix in the right hand side is defined as the absolute value of its eigenvalues. In other words, if  $A$  is a diagonalizable matrix  $A = U D U^{-1}$ , then

$$|A| = U |D| U^{-1} = U \begin{pmatrix} |d_1| & 0 & \cdots \\ 0 & |d_2| & \cdots \\ \vdots & & \ddots \end{pmatrix} U^{-1}. \quad (25)$$

Note that, for translationally invariant interaction we consider, the interaction matrices  $M$  are of Toeplitz type, i.e.  $(M_{\text{fw/bw}})_{ij} = (M_{\text{fw/bw}})_{|i-j|}$ .

Now on we focus on the limit where the number of modes is large ( $\nu \gg 1$ ). Therefore, one can impose the periodic boundary condition in the mode space, without changing the results. In this case the interaction matrices are circulant,  $M_{|\nu-i-j|} = M_{|i-j|}$ , and can be easily diagonalized<sup>34</sup>.

We adopt a  $g$ -ology type notations and model the interaction by  $g_2$  and  $g_4$  components

$$(M_{\text{fw}})_{|i-j|} = v_F \delta_{ij} + g_4(|i-j|), \quad (26)$$

$$(M_{\text{bw}})_{|i-j|} = -g_2(|i-j|). \quad (27)$$

Here the distance dependent  $g_4(i)$  accounts for the forward interactions between electron densities of the same chirality at distance  $i$ , while  $g_2(i)$  parameterizes the backward interactions of densities of opposite chiralities. The scaling dimension  $\Delta_\ell$ , defined in Eq.(24) with  $\mathbf{m}$  given by Eq.(23) is

$$\Delta_\ell = \frac{1}{\nu} \sum_{k=0}^{\nu-1} \frac{1 - G(k) \cos(2\pi k \ell / \nu)}{\sqrt{1 - G^2(k)}}. \quad (28)$$

The function  $G(k)$ , is determined by the interaction parameters  $g_{2,4}$

$$G(k) = \frac{\tilde{g}_2(k)}{v_F + \tilde{g}_4(k)}. \quad (29)$$

Here  $\tilde{g}_{2,4}(k) = \sum_{j=1}^{\nu} \cos(2\pi j k / \nu) g_{2,4}(j)$  is the cosine transform of  $g_{2,4}(r)$ . The condition of  $|G(k)| < 1$  follows from positivity of matrix  $\mathcal{M}$ . With the scaling of disorder operators at hand we can analyze their behavior under renormalization.

We now focus on a finite range interaction. One can easily show that the scattering processes between distant modes are less effective for the localization than backscattering between close ones. For the model of isotropic interaction  $g_4(r) = g_2(r) = g \exp(-r^2/R^2)$  the scattering between the distant modes (12) is irrelevant for

$$\sqrt{\frac{g}{2v_F}} > \frac{1}{\pi} \frac{R}{d}. \quad (30)$$

Here we assumed that  $|i-j| = \ell \gg 1$  and  $g/v_F \gg 1$ .

The scattering between the nearest modes ( $\ell = 1$ ) imposes more stringent conditions on the interaction constants  $\lambda_a^b$ , as shown in Fig.4. In particular, the conducting phase is stable only for the nearly symmetric interaction. For the fixed values of interaction strength the localization is enhanced by increasing the interaction radius. In other words, strong and short range interaction most efficiently drives the system towards the conducting phase.

### III. SUMMARY

To summarize, we have studied the localization of the edge modes in TIs with TR symmetry. We find that a combination of TR symmetry and zero bias anomaly changes the scaling dimensions of scattering operators. This notably affects the phase diagram. For a sufficiently strong values of interaction the zero temperature fixed point is a conductor with a number of edge modes that

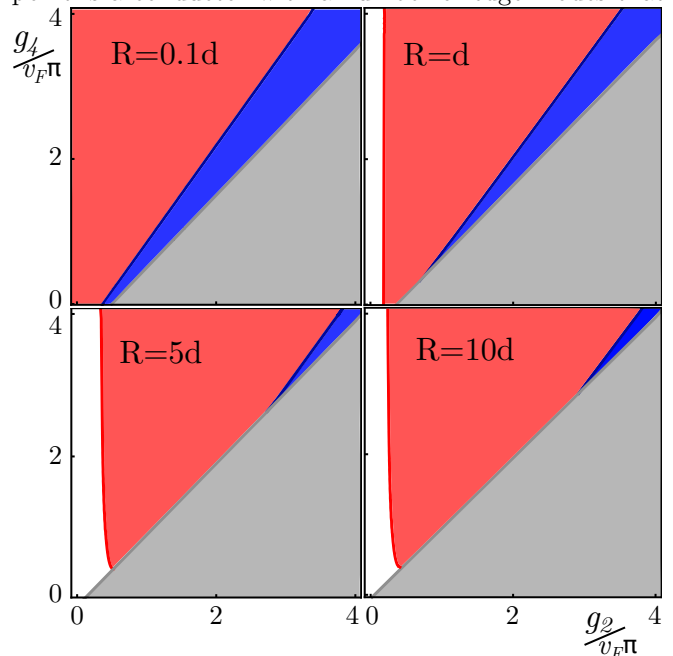


FIG. 4. (color online).  $(g_2, g_4)$  phase diagram  $\nu \gg 1$ , finite ranges interaction  $g_{2,4}(n) = g_{2,4} \exp(-(\frac{nd}{R})^2)$ . Panels correspond to the different values of the interaction radius  $R$ . Color code is the same as in Fig.2

are stable against TR disorder. This holds also for the even fillings, where the non-interacting system is equivalent to a trivial insulator. We have analyzed the problem in several limiting cases, for the single impurity and random disorder, short and long range interaction, for a variety of filling fractions  $\nu$ . We have computed the boundaries of the conducting phase in all these cases. For intermediate values of interaction electric conductivity is a non-monotonous function of temperature, due to interplay of single and two electron scattering processes.

The authors acknowledge discussions with E. Berg, Y. Gefen, I. V.Gornyi, N. Kainaris, A.D. Mirlin, I.V. Protodopov, E. Sela. This work was supported by GIF and ISF.

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